Exercise 10

Solve the differential equation.

$$3y'' + 4y' - 3y = 0$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad y' = re^{rx} \quad \rightarrow \quad y'' = r^2 e^{rx}$$

Plug these formulas into the ODE.

$$3(r^2 e^{rx}) + 4(re^{rx}) - 3(e^{rx}) = 0$$

Divide both sides by e^{rx}.

$$3r^2 + 4r - 3 = 0$$

Solve for r.

$$r = \frac{-4 \pm \sqrt{16 - 4(3)(-3)}}{2(3)} = \frac{-4 \pm \sqrt{52}}{6} = -\frac{2}{3} \pm \frac{\sqrt{13}}{3}$$

$$r = \left\{-\frac{2}{3} - \frac{\sqrt{13}}{3}, -\frac{2}{3} + \frac{\sqrt{13}}{3}\right\}$$

Two solutions to the ODE are $e^{(-2/3-\sqrt{13}/3)x}$ and $e^{(-2/3+\sqrt{13}/3)x}$. By the principle of superposition, then,

$$y(x) = C_1 e^{(-2/3-\sqrt{13}/3)x} + C_2 e^{(-2/3+\sqrt{13}/3)x},$$

where C_1 and C_2 are arbitrary constants.